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Dust Charge Fluctuation-Induced Shock Wave Formation in Dusty Plasma: A Theoretical Investigation

Ajay Paul
Department of Basic Sciences, Asian College of Engineering and Technology, Chennai, Tamil Nadu, India
E-Mail: ajaypaul34@gmail.com
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Abstract **- The nonlinear propagation of ion-acoustic waves (IAWs) in a dusty plasma consisting of Boltzmann-distributed electrons and ions, mobile charge-fluctuating negative dust, and stationary charge-fluctuating negative dust has been theoretically investigated. It has been shown that dust charge fluctuation is a source of dissipation and is responsible for the formation of DA shock waves in such a dusty plasma. The basic features of the DA shock waves have been identified in this investigation, which could be useful in understanding the properties of localized space dusty plasmas. A new laboratory experiment has been proposed to identify the basic features of the dust-acoustic shock waves predicted in this theoretical investigation.**
Keywords: **Charge Fluctuation, Linear Dust-Acoustic Waves, Dusty Plasmas, Shock Waves.**

I. INTRODUCTION

Wave propagation in dusty plasmas has received considerable attention in recent years because of its vital role in understanding different types of collective processes in space environments, namely, the lower and upper mesosphere, cometary tails, planetary rings, planetary magnetosphere, interplanetary spaces, interstellar media, etc. [1]-[6]. Dusty plasmas also have notable applications in laboratory devices [7]-[10]. The consideration of charged dust grains in plasmas not only modifies the existing plasma wave spectra [11]-[13], but also introduces a number of novel eigenmodes, such as the dust ion-acoustic (DIA) waves, the dust-acoustic (DA) waves, the dust lower-hybrid (DLH) waves, the dust lattice (DL) waves, etc [14]-[18]. Most studies of dusty plasmas have considered dust as negatively charged grains in addition to electrons and positively charged ions as the plasma species [4]-[6], [19]-[24]. It has been found that there are some plasma systems, particularly in space plasma environments, namely, cometary tails [1]-[3], [25], [26], upper mesosphere [27], Jupiter's magnetosphere [28], etc. where positively charged dust grains play significant roles. There are three main mechanisms by which the dust grains in the plasma systems mentioned above can be positively charged. These mechanisms are the following: (i) photoemission in the presence of a flux of ultraviolet (UV) photons; (ii) thermionic emission induced by radiative heating; and (iii) secondary emission of electrons from the surface of the dust grains. In this paper, we have considered a dusty plasma containing mobile charge-fluctuating negative dust, stationary charge-fluctuating negative dust, Boltzmann-distributed electrons and ions, and have studied the linear propagation of DA waves. This paper is organized as follows. The basic equations describing our dusty plasma model are presented in Section II. The dispersion relation is derived in Section III. The dispersion properties of the DA wave mode are analyzed numerically in the dispersion properties of the DA wave mode in Section IV, where I have seen that negative dust-charge fluctuation is a source of linear growth instability. Finally, a brief discussion is given in Section V.

II. GOVERNING EQUATIONS

We consider an unmagnetized, collisionless dusty plasma system consisting of charge-fluctuating positively charged mobile dust, stationary charge-fluctuating negative dust and Boltzmann-distributed electrons and ions. Thus, at equilibrium, we have where () is the equilibrium electron (ion) number density, is the negative dust number density, is the positive dust number density, is the equilibrium charge state of the positive dust component, is the equilibrium charge state of the negative dust component. The dynamics of the DA waves of such a dusty plasma system in one-dimensional form is given by

$$\frac{\partial n_d^+}{\partial t}+\frac{\partial}{\partial x}(n_d^+u_d^+)=0 \quad (1)$$

$$\frac{\partial u_d^+}{\partial t}+u_d^+\frac{\partial u_d^+}{\partial x}=-\frac{z_d^+e}{m_d}\frac{\partial \varphi}{\partial x} \quad (2)$$

$$\frac{\partial^2\varphi}{\partial x^2}=4\pi e\left[n_e-n_i-z_d^+n_d^+ + z_d^-n_d^-\right] \quad (3)$$

where n_j is the number density of the plasma species j (j = i for ions and e for electrons), $n_d^+(n_d^-)$ is the number density of positive (negative) dust. $u_d^+(u_d^-)$ is the positive (negative) dust fluid speed. $z_d^+(z_d^-)$ is the charge state of the positive (negative) dust component. φ is the

electrostatic wave potential. The electron and ion densities are assumed to follow the Boltzmann distribution:

$$n_e = n_{eo}\exp\left(\frac{e\varphi}{k_B T_e}\right) \quad (4)$$

$$n_i = n_{io}\exp\left(-\frac{e\varphi}{k_B T_i}\right) \quad (5)$$

where k_B is the Boltzmann constant and T_e is the electron temperature, and T_i is the ion temperature. We assume that dust is charged by the photoemission current $\left(I_P^+\right)$, the thermionic emission current $\left(I_t^+\right)$ and the electron absorption current $\left(I^-\right)$, the electron current for negatively charged dust $\left(I_e\right)$, the ion current for negative charge dust $\left(I_i\right)$ only. All other charging processes are neglected. The charge state z_d^+ component is not constant, but varies according to the following equations:

$$\frac{\partial z_d^+}{\partial t} + u_d^+\frac{\partial z_d^+}{\partial t} = \frac{I_P^+ + I_t^+ + I^-}{e} \quad (6)$$

$$\frac{\partial z_d^-}{\partial t} = -\left(\frac{I_e + I_i}{e}\right) \quad (7)$$

where

$$I_p^+ = \pi r_d^2 e\, JY \exp\left(-\frac{z_d^+ e^2}{k_B r_d T_{ph}}\right) \quad (8)$$

$$I_t^+ = 2\pi r_d^2 e\left(\frac{m_e k_B T_p}{2\pi\hbar^2}\right)^{\frac{3}{2}}\left(\frac{8k_B T_p}{\pi m_e}\right)^{\frac{1}{2}}\left(1+\frac{z_d^+ e^2}{k_B r_d T_p}\right)\times\exp\left(-\frac{z_d^+ e^2}{k_B r_d T_p}-\frac{W_e}{k_B T_p}\right) \quad (9)$$

$$I^- = -\pi r_d^2 e\, n_{eo}\, e^{\frac{e\varphi}{k_B T_P}}\left(\frac{8k_B T_P}{\pi m_e}\right)^{\frac{1}{2}}\left(1+\frac{z_d^+ e^2}{k_B r_d T_p}\right) \quad (10)$$

$$I_e = -4\pi r_d^2\, n_{eo} e^{\frac{e\varphi}{k_B T_e}} e\left(\frac{k_B T_e}{2\pi m_e}\right)^{\frac{1}{2}}\exp\left(-\frac{z_d^- e^2}{k_B r_d T_e}\right) \quad (11)$$

$$I_i = 4\pi r_d^2\, n_{io} e^{-\frac{e\varphi}{k_B T_i}} e\left(\frac{k_B T_i}{2\pi m_i}\right)^{\frac{1}{2}}\left(1+\frac{z_d^- e^2}{k_B r_d T_i}\right) \quad (12)$$

where $\hbar$ is the Planck's constant, T_{ph} is the photon temperature, W_e is the work function, J is the UV photon flux, Y is the yield of photons (typical values of W_e, J and Y are 2.2 eV, 5.0×10^{14} photons/cm^2/s, and 0.1, respectively), and r_d is the dust radius. Now, using $q_d = +z_d^+ e$ (where the charge state z_d is the number of electrons residing on the dust grain surface). We introduce the following normalized variables:

$N_d^+ = n_d^+/n_{do}^+$, $U_d^+ = u_d^+/C_d$, $\Phi = e\varphi/k_B T_e$,
$Z_d^- = z_d^-/z_{do}^-$ $Z_d^+ = z_d^+/z_d^+$, $Z_d^- = z_d^-/z_{do}^-$, $X = x/\lambda_{Dd}$,
$T = t\,\omega_{pd}$,
$\lambda_{Dd} = \left(k_B T_e/4\pi z_{do}^+ n_{do}^+ e^2\right)^{1/2}$, $C_d = \left(z_{do}^+ k_B T_e/m_d\right)^{1/2}$
and $\omega_{pd} = \left(4\pi z_{do}^{+2} n_{do}^+ e^2/m_d\right)^{1/2}$.

Equations (1)–(7) can be reduced to

$$\frac{\partial N_d^+}{\partial T} + \frac{\partial}{\partial X}\left(N_d^+ U_d^+\right) = 0 \quad (13)$$

$$\frac{\partial U_d^+}{\partial T} + U_d^+\frac{\partial U_d^+}{\partial X} = -Z_d^+\frac{\partial \Phi}{\partial X} \quad (14)$$

$$\frac{\partial^2\Phi}{\partial X^2} = \left(1+\mu_i-\gamma\right)e^{\Phi} - \mu_i e^{-\sigma\Phi} - Z_d^+ N_d^+ + \gamma Z_d^- \quad (15)$$

$$\frac{\partial Z_d^+}{\partial T} + U_d^+\frac{\partial Z_d^+}{\partial X} = \mu^+\left[Pe^{-\alpha Z_d^+} + Q\left(1+\beta Z_d^+\right)e^{-\beta Z_d^+} - \mathrm{Re}^{\Phi}\left(1+\beta Z_d^+\right)\right] \quad (16)$$

$$\frac{\partial Z_d^-}{\partial T} = \mu^-\left[X_e e^{\Phi-\alpha_e Z_d^-} - X_i\left(1+\alpha_i Z_d^-\right)e^{-\sigma\Phi}\right] \quad (17)$$

Where $\mu^- = \dfrac{4\pi r_d^2}{z_{do}^- \omega_{pd}}$ $\mu^+ = \dfrac{\pi r_d^2}{z_{do}^+ \omega_{pd}}$, $\alpha_e = \dfrac{z_{do}^- e^2}{r_d k_B T_e}$,

$\alpha_i = \dfrac{z_{do}^- e^2}{r_d k_B T_i}$

$\alpha = \dfrac{z_{do}^+ e^2}{r_d k_B T_{ph}}$, $\beta = \dfrac{z_{do}^+ e^2}{r_d k_B T_p}$, $\gamma = \dfrac{z_{do}^- n_{do}^-}{z_{do}^+ n_{do}^+}$,

$\mu_e = \dfrac{n_{eo}}{z_{do}^+ n_{do}^+}$, $\mu_i = \dfrac{n_{io}}{z_{do}^+ n_{do}^+}$,

$\sigma = \dfrac{T_e}{T_i}$, $X_e = n_{eo}\left(\dfrac{k_B T_e}{2\pi m_e}\right)^{1/2}$, $X_i = n_{io}\left(\dfrac{k_B T_i}{2\pi m_i}\right)^{1/2}$,

P = JY,

$$Q = 2\left(\frac{m_e k_B T_P}{2\pi\hbar^2}\right)^{3/2}\left(\frac{8k_B T_P}{\pi m_e}\right)^{1/2} e^{\frac{-W_e}{k_B T_P}},$$

$$R = n_{eo}\left(\frac{8k_B T_p}{\pi m_e}\right)^{1/2}.$$

III. DERIVATION OF THE LINEAR DISPERSION RELATION

To derive a dynamical equation for the linear propagation of the DA shock waves in a dusty plasma, we first express the

dependent variables N_d^+, $U_{d,}^+$ Φ, Z_d^+, and Z_d^- in terms of their equilibrium and perturbed parts as

$$N_d^+ = 1 + \in N_d^{+(1)} + \in^2 N_d^{+(2)} + \text{---------} \quad (18)$$

$$U_d^+ = \in U_d^{+(1)} + \in^2 U_d^{+(2)} + \text{---------} \quad (19)$$

$$Z_d^+ = 1 + \in Z_d^{+(1)} + \in^2 Z_d^{+(2)} + \text{---------} \quad (20)$$

$$Z_d^- = 1 + \in Z_d^{+(1)} + \in^2 Z_d^{+(2)} + \text{---------} \quad (21)$$

$$\Phi = \in \Phi^{(1)} + \in^2 \Phi^{(2)} + \text{----------} \quad (22)$$

Substituting Eqs. (18)–(22) into Eqs. (13)–(17) and collecting terms of different orders, we obtain

$$\frac{\partial N_d^{+(1)}}{\partial T} + \frac{\partial U_d^{+(1)}}{\partial X} = 0 \quad (23)$$

$$\frac{\partial U_d^{+(1)}}{\partial T} = -\frac{\partial \Phi^{(1)}}{\partial X} \quad (24)$$

$$\frac{\partial^2 \Phi}{\partial X^2} = (1 + \mu_i - \gamma)\Phi^{(1)} + \mu_i \sigma \Phi^{(1)} - N_d^{+(1)} - Z_d^{+(1)} + \gamma Z_d^{-(1)} \quad (25)$$

$$\frac{\partial Z_d^{+(1)}}{\partial T} = \mu^+ [(Q\beta^2 - P\alpha) Z_d^{+(1)} - R(1+\beta)\Phi^{(1)}] \quad (26)$$

$$\frac{\partial Z_d^{-(1)}}{\partial T} = \mu^- [(-\alpha_e X_e - \alpha_i X_i) Z_d^{-(1)} - (X_e + X_e \alpha_e - X_i \sigma - X_i \sigma \alpha_i)\Phi^{(1)}] \quad (27)$$

Assuming that all perturbed quantities are proportional to $\exp(-i\omega T + ikX)$, i.e., taking $\partial / \partial T \to -i\omega$ and $\partial / \partial X \to ik$, where ω and k are the wave angular frequency and the propagation constant, respectively, in Eqs. (23)–(27), we obtain

Using Eqs. (30)–(34), we can eliminate $N_d^{+(2)}$, $U_d^{+(2)}$, $Z_d^{+(2)}$, $Z_d^{-(2)}$ and $\Phi^{(2)}$, and finally obtain the following equation:

$$\frac{\partial \Phi^{(1)}}{\partial \tau} + A\Phi^{(1)} \frac{\partial \Phi^{(1)}}{\partial \xi} = C \frac{\partial^2 \Phi^{(1)}}{\partial \xi^2} \quad (35)$$

where the nonlinear coefficient A and the dissipation coefficient C are given by

$$A = \frac{B'}{A'} \quad (36)$$

$$C = -\frac{C'}{A'} \quad (37)$$

$$A' = -\frac{2}{V_0^3} \quad (38)$$

$$B' = (1 + \mu_i - \gamma) - \sigma^2 \mu_i - \frac{3f}{V_0^2} + \frac{f^2\left(P\alpha^2 - Q\beta^2 + \frac{3}{2}Q\beta^3\right)}{P\alpha^2 - Q\beta^2 + \frac{3}{2}Q\beta^3 - P\alpha - R\beta} - \frac{2R\beta f + R + R\beta}{P\alpha^2 - Q\beta^2 + \frac{3}{2}Q\beta^3 - P\alpha - R\beta} - \frac{3}{V_0^4} + \frac{2\gamma\left(\frac{1}{2}X_e - \frac{1}{2}X_e\alpha_e - X_e\alpha_e\rho + \frac{1}{2}\alpha_e^2 X_e\rho^2 + \alpha_e^2 X_e\rho - \frac{1}{2}\sigma^2 X_i + \frac{1}{4}\alpha_e^2 X_e - \frac{1}{2}\sigma^2 X_i\alpha_i + \sigma^2 X_i\alpha_i\right)}{\alpha_i X_i - X_e\alpha_e^2 + X_e\alpha_e} \quad (39)$$

$$C' = \frac{V_0 f}{\mu^+\left(P\alpha^2 - Q\beta^2 + \frac{3}{2}Q\beta^3 - P\alpha - R\beta\right)} + \frac{V_0\rho\gamma}{\mu^-\left(\alpha_i X_i - X_e\alpha_e^2 + X_e\alpha_e\right)} \quad (40)$$

Equation (35) is the well-known Burgers equation describing the nonlinear propagation of DA shock waves in the dusty plasma under consideration. It is evident from Eqs. (35) and (37) that the dissipative term, i.e., the right-hand side of Eq. (35), is due to the presence of charge-fluctuating dust.

IV. NUMERICAL ANALYSIS

We now seek the stationary shock-wave solution of Eq. (35) by introducing the variables $\varsigma = \xi - U_0\tau'$ and $\tau' = \tau$, where U_0 is the shock wave speed (in the reference frame) normalized by C_d, ς is normalized by λ_{Dd}, and τ is normalized by ω_{pd}^{-1}. This leads us to write (35), under the steady state condition ($\partial / \partial \tau = 0$), as

$$-U_0 \frac{\partial \Phi^{(1)}}{\partial \varsigma} + A\Phi^{(1)} \frac{\partial \Phi^{(1)}}{\partial \varsigma} = C \frac{\partial^2 \Phi^{(1)}}{\partial \varsigma^2} \quad (41)$$

It can be shown [36], [37] that Eq. (41) describes shock waves whose speed U_0 (in the reference frame) is related to the extreme values $\Phi^{(1)}(-\infty)$ and $\Phi^{(1)}(\infty)$ by $\Phi^{(1)}(-\infty) - \Phi^{(1)}(\infty) = 2U_0 / A$. Thus, provided that $\Phi^{(1)}$ is bounded at $\varsigma = \pm\infty$, the shock wave solution of (41) can be written as

$$\Phi^{(1)} = \Phi_0[1 - \tanh(\varsigma / \Delta)] \quad (42)$$

where

$$\Phi_0 = U_0 / A \quad (43)$$

$$\Delta = 2C / U_0 \quad (44)$$

are the height and thickness, respectively, of the shock waves moving with the speed U_0. It is obvious from (41) to (44) that the shock waves are due to the presence of the charge-fluctuating dust, and the shock structures are associated with the negative potential $(A<0)$ as well as with positive potential $(A>0)$.

To find the parametric regimes for which positive and negative shock wave (potential) profiles exist, we have numerically analyzed A and obtained two-dimensional A = 0 curves for $\gamma = 0.2$ to 0.6 and $\mu_i = 0$ to 3.8. The A = 0 curve is shown in Fig. 1.

It shows that we can have positive shock wave (potential) profiles for the parameters whose values lie above the A = 0 curve and negative shock wave (potential) profiles for the parameters whose values lie below the A = 0 curve. These are shown in Figs. 2 and 3. Figs. 2 and 3 show the positive and negative shock potential profiles, respectively.

V. DISCUSSION

The present theoretical analysis demonstrates that fluctuations in dust-grain charge play an essential role in shaping nonlinear electrostatic shock structures in the plasma model considered. In particular, the reduction of the governing equations to the Burgers equation [Eq. (35)] identifies a dissipative contribution associated with time-dependent dust charging. This indicates that dust-charge variation is not merely a modification of the equilibrium charge balance; within the assumptions of the model, it provides the mechanism that allows a steepened nonlinear wave profile to develop into a finite-width shock.

The balance between nonlinear steepening and charge-fluctuation-induced dissipation determines the character of the stationary shock profile. The nonlinear coefficient A controls the polarity of the electrostatic potential structure, whereas the dissipative coefficient C determines its smoothing and spatial extent. Consequently, changing the plasma parameters can alter the sign of the nonlinear response and permit shocks with either positive or negative potentials.

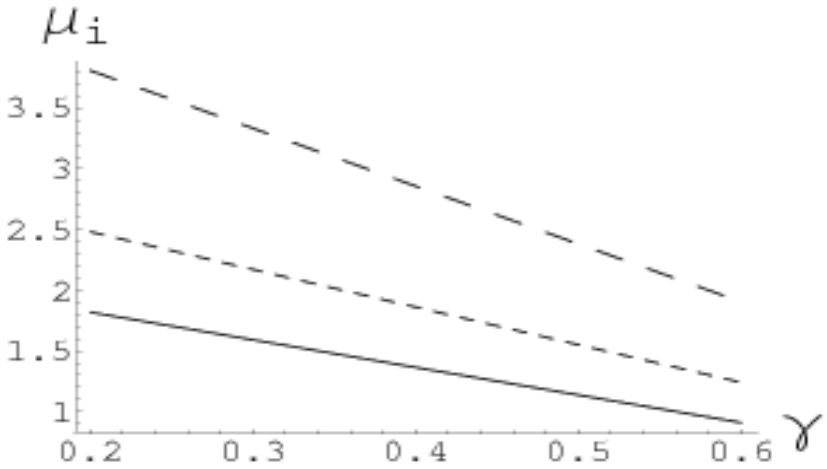

Fig.1 Showing A = 0 (vs.) curves for the parameters P = 5.0 .

Q = 1.93×10^{28} $cm^{-2}s^{-1}$. R = $2.48\times10^{28} cm^{-2}s^{-1}$ with

$\alpha_i = 4.77$, $\beta = 38.6$.

σ =1.2(Solid Curve). σ =1.15(Dotted curve) ,

σ =1.1(dashed curve).

The A = 0 boundary plotted in Fig. 1 therefore separates the parameter regimes supporting the two types of shock profiles. Close to this boundary, the leading nonlinear coefficient becomes small, and the Burgers-equation approximation should be interpreted with particular care. Specifically, Fig. 1 presents the A = 0 boundary as a function of the plasma parameters γ and μ_i for three values of the electron-to-ion temperature ratio, σ = 1.2 (solid curve), 1.15 (dotted curve), and 1.1 (dashed curve). The different boundary curves illustrate how the parameter region separating positive- and negative-potential shocks depends on σ.

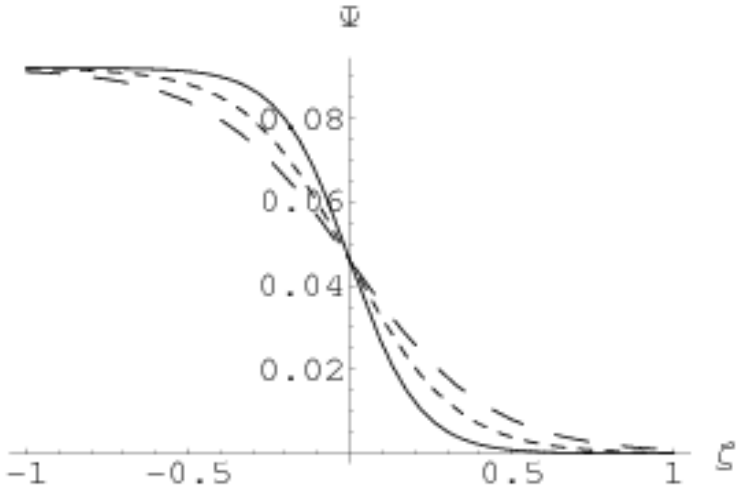

Fig.2 Showing positive potential (vs.) curves for the parameters

$P = 5.0 \times 10^{13}\ cm^{-2}$. $Q = 1.93\times10^{28}\ cm^{-2}\cdot s^{-1}$. $R = 2.48\times10^{28} cm^{-2}\cdot s^{-1}$ with $\alpha_i = 4.77$, $\beta = 38.6$. $\mu_i = 4.5$ (Solid Curve). $\mu_i = 5.6$ (Dotted curve), $\mu_i = 7.5$ (dashed curve).

The positive- and negative-potential profiles presented in Figs. 2 and 3 illustrate these distinct shock regimes. They show that the predicted polarity is sensitive to the chosen set of plasma parameters rather than being a fixed property of the dusty plasma. The analytical expressions for the stationary solution [Eqs. (42)–(44)] further indicate that, for fixed model coefficients, the normalized shock height increases with shock speed, while the normalized shock thickness decreases as shock speed increases. Thus, faster shocks are expected to be taller and more sharply localized, reflecting the balance between stronger nonlinear steepening and the available dissipation.

In Fig. 2, the positive-potential shock profiles are plotted for μ_i = 4.5, 5.6, and 7.5, represented by solid, dotted, and dashed curves, respectively. In contrast, Fig. 3 displays negative-potential shock profiles for μ_i = 0.5, 1.0, and 1.3, using the same sequence of line styles. These figures illustrate how the shock potential profiles vary with μ_i within the respective positive- and negative-potential regimes.

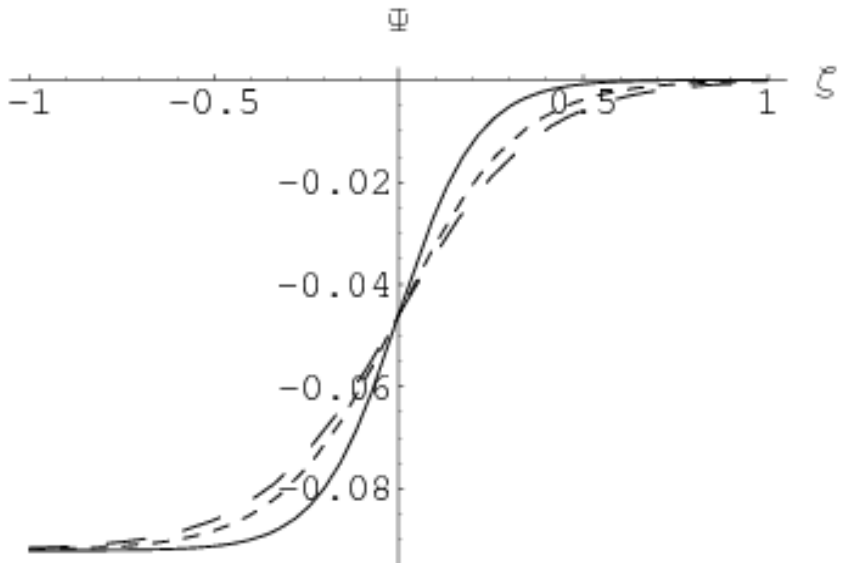

Fig.3 Showing negative potential (vs.) curves for the parameters

$P = 5.0 \times 10^{13}\ cm^{-2}$. $Q = 1.93\times10^{28}\ cm^{-2}\cdot s^{-1}$. $R = 2.48\times10^{28} cm^{-2}\cdot s^{-1}$ with $\alpha_i = 4.77$, $\beta = 38.6$. $\mu_i = 0.5$ (Solid Curve). $\mu_i = 1.0$ (Dotted curve), $\mu_i = 1.3$ (dashed curve).

These results extend the physical interpretation of nonlinear wave propagation in dusty plasmas by emphasizing the role of dynamical dust charging. They are potentially relevant to regions such as the terrestrial mesosphere, where charged dust particles have been observed [27]. Nevertheless, the present results follow from an idealized, one-dimensional, unmagnetized, and collisionless model with Boltzmann-distributed electrons and ions and the charging processes specified in Section II. The analysis does not establish that the predicted shock profiles occur under all mesospheric conditions. Additional investigation, including quantitative parameter constraints and comparisons with observations or more comprehensive simulations, would be required to assess their applicability in a specific environment.

VI. CONCLUSION

We have studied the nonlinear propagation of DA waves in an unmagnetized dusty plasma containing Boltzmann-distributed electrons and ions, mobile charge-fluctuating negative dust and stationary charge-fluctuating negative dust. We have shown how the basic features of the nonlinear IA waves are modified by the presence of the charge-fluctuating dust in dusty plasmas. The results of this investigation can be summarized as follows. Dust charge fluctuation is a source of dissipation and is responsible for the formation of IA shock waves in the dusty plasma. The shock structures are associated with the negative potential (A<0) as well as positive potential (A<0). The height (normalized by $k_B T_e/e$) of the potential structures in the form of the shock waves is directly proportional to the shock speed U_0, and the thickness (normalized by λ_{Dd}) of these shock structures is inversely proportional to the shock speed U_0. The parametric regimes for the existence of positive as well as negative shock structures are shown in Fig. 1. Figs. 2 and 3 show the positive and negative shock potential profiles of shock waves respectively. The parameters chosen for the numerical analysis are relevant to plasma in the mesosphere [27]. We stress that the results of the present investigation could be useful in understanding the properties of localized DA waves of dusty plasmas in the mesosphere.

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