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Common Fixed Point Theorems Under Strict Contractive Conditions in Fuzzy Metric Spaces

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Abstract **-Fuzzy sets originally introduced Zadeh. Using Concept of fuzzy sets, various theories regarding giving new concepts of fuzzy metric spaces were considered by various authors. After that, many authors have studied fixed point theory in such spaces and proved various fixed point theorems in such spaces. In 1975, Kramosil*et al.* have introduced the concept of fuzzy metric spaces. In this paper, using concept of fuzzy metric space given by Kramosil et al., we prove common fixed point theorems for weakly compatible mappings satisfying strict contractive condition in fuzzy metric spaces by using property (E.A). Our proved results generalize known fixed point theorems in literature. We prove two common fixed point theorems in this paper, one for four maps and other one for two maps. Mathematics subject classification: 54E40, 54E35, 54H25.**
Keywords:**FuzzyMetric Space, Compatible Mappings, Weakly Compatible Mappings**

I. INTRODUCTION

In 1986, Jungck [2] introduced the concept of Compatible mapping and proved some common fixed point theorems of compatible mappings in metric space. In 1997, Pant et al [7] gave two common fixed point theorems of non-compatible mappings under strict contractive conditions by using the notion of R-Weak commutativety. By using this property, some common fixed point theorems under strict contractive conditions in metric spaces have been given.

The notion of fuzzy sets was introduced by Zadeh. Various concepts of fuzzy metric spaces were considered by various authors in [1-10]. Many authors have studied fixed point theory in fuzzy metric spaces. The authors [1-10] have proved fixed point theorems in fuzzy (probabilistic) metric spaces. Kramosil*et al.* (1975) have introduced the concept of fuzzy metric spaces in different ways [1-10].

II. PRELIMINARIES

The concept of triangular norms (t-norms) is originally introduced by Menger in study of statistical metric spaces.

Definition 1[9] A binary operation *: $[0,1]\times[0,1] \rightarrow [0,1]$ is continuous t-norm if * satisfies the following conditions

1. * is commutative and associative;
2. * is continuous;
3. $a * 1 = a$ for all a in [0,1];
4. $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0,1]$.

Examples of t-norms are: $a*b = min\{a, b\}$, $a*b = ab$ and $a*b = max\{a+b-1, 0\}$.

Kramosil*et al.* (1975)[3] introduced the concept of fuzzy metric spaces as follows:

Definition 2[3] A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, *is a continuous t-norm, and M is fuzzy sets on $X^2\times[0, \infty)$ satisfying the following conditions for all $x, y, z \in X$ and $s, t > 0$,

1. $M(x, y, 0) = 0$;
2. $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$;
3. $M(x, y, t) = M(y, x, t)$;
4. $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$;
5. $M(x, y, .) : [0, \infty) \rightarrow [0,1]$ is left continuous.

Then $(X, M, *)$ is called a fuzzy metric space on X. The function $M(x, y, t)$ denote the degree of nearness between x and y w.r.t. t respectively.

Remark 3[3] In fuzzy metric space $(X, M, *)$, $M(x, y, .)$ is non-decreasing for all $x, y \in X$.

Definition 4 [3] Let $(X, M, *)$ be a fuzzy metric space. Then a sequence $\{x_n\}$ in X is said to be
(a) convergent to a point $x \in X$ if, for all $t > 0$,
$lim_{n\rightarrow\infty} M(x_n, x, t) = 1$.
(b) Cauchy sequence if, for all $t > 0$ and $p > 0$,
$lim_{n\rightarrow\infty} M(x_{n+p}, x_n, t) = 1$.

Definition 5[3] A fuzzy metric space $(X, M, *)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 6 [10] A pair of self-mappings (A, S) of a fuzzy metric space $(X, M, *)$ is said to be commuting if $M(ASx, SAx, t) = 1$ for all $x \in X$.

Definition 7 [10] A pair of self-mappings (A, S) of a fuzzy metric space $(X, M, *)$ is said to be weakly commuting if $M(ASx, SAx, t) \ge M(Ax, Sx, t)$ for all $x \in X$ and $t > 0$.

Definition 8[2] A pair of self-mappings (*A*, *S*) of a fuzzy metric space $(X,M,*)$ is said to be compatible if $lim_{n\to\infty}M(ASx_n, SAx_n, t) = 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in *X* such that $lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = u$ for some $u\in X$.

Definition 9[2] Let $(X,M,*)$ be a fuzzy metric space. *A* and *S* be self-maps on *X*. A point $x\in X$ is called a coincidence point of *A* and *S* iff $Ax = Sx$. In this case, $w = Ax = Sx$ is called a point of coincidence of *A* and *S*.

Definition 10[2] A pair of self-mappings (*A*, *S*) of a fuzzy metric space $(X,M,*)$ is said to be weakly compatible if they commute at the coincidence points i.e., if $Au = Su$ for some $u \in X$, then $ASu = SAu$. It is easy to see that two compatible maps are weakly compatible but converse is not true.

Definition 11[10] A pair of self-mappings (*A*, *S*) of a fuzzy metric space $(X,M,*)$ is said to be pointwise *R*-weakly commuting if given $x\in X$, there exist $R > 0$ such that $M\left(ASx, SAx, t\right)\geq M\left(Ax, Sx, \frac{t}{R}\right)$ for all $t > 0$.

Clearly, every pair of weakly commuting mappings is pointwise *R*-weakly commuting with $R = 1$.

Definition 12[7] Two mappings *A* and *S* of a fuzzy metric space $(X,M,*)$ will be called reciprocally continuous if $ASu_n \to Az, SAu_n \to Sz$, whenever $\{u_n\}$ is a sequence such that $Au_n \to z, Su_n \to z$ for some $z\in X$.

If *A* and *S* are both continuous, then they are obviously reciprocally continuous but converse is not true.

Lemma 1 [3] Let $\{u_n\}$ is a sequence in a fuzzy metric space $(X,M,*)$. If there exists a constant $h\in(0,1)$ such that $M(u_n,u_{n+1},ht)\geq M(u_{n-1},u_n,t)$, $n = 1, 2, 3, \ldots$

Then $\{u_n\}$ is a Cauchy sequence in *X*.

Definition 13 Let *A* and *S* be mappings from a fuzzy metric space $(X, \mathcal{M},*)$ into itself. Then the mappings are said to be weak compatible if they commute at their coincidence point, that is $Ax=Sx$ implies that $ASx=SAx$.

Definition 14 Let *A* and *B* be two self-mappings of a fuzzy metric space $(X, \mathcal{M}, *)$. We say that *A* and *B* satisfy the property (E-A), if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} \mathcal{M}(Ax_n, u, t) = \lim_{n\to\infty}\mathcal{M}(Bx_n, u, t) = 1$ for same $u\in X$ and $t>0$.

Example 1

Let X=R and $\mathcal{M}(x, y, t) = \frac{t}{t+|x-y|}$ for every $x, y\in X$ and $t>0$. Let A and B be defined $Ax = 2x+1$, $Bx = x+2$, consider the sequence $x_n = \frac{1}{n}+1$, n=1, 2, …

Thus we have $\lim_{n\to\infty}\mathcal{M}(Ax_n, 3,3, t) = \lim_{n\to\infty}\mathcal{M}(Bx_n, 3,3, t) = 1$ for every $t>0$.

Then A and B satisfying in the property (E-A).

In the next example we show that there are some mappings that have not property (E-A).

Example 2

Let X=R and $\mathcal{M}(x, y, t) = \frac{t}{t+|x-y|}$ for every $x, y\in X$ and $t>0$.
Let $Ax=x+1$ and $Bx=x+2$, if sequence $\{x_n\}$ there exist such that $\lim_{n\to\infty}\mathcal{M}(Ax_n, u, u, t) = \lim_{n\to\infty}\mathcal{M}(Bx_n, u, u, t) = 1$ for some $u\in X$.
Therefore $\lim_{n\to\infty}\mathcal{M}(Ax_n, u, u, t) = \lim_{n\to\infty}\mathcal{M}(x_n + 1, u, u, t) = \lim_{n\to\infty}\mathcal{M}(x_n, u - 1, u - 1, t) = 1$ and $\lim_{n\to\infty}\mathcal{M}(Bx_n, u, u, t) = \lim_{n\to\infty}\mathcal{M}(x_n + 2, u, u, t) = \mathcal{M}(x_n, u - 2, u - 2, t) = 1$. We conclude that, $x_n\to u-1$ and $x_n\to u-2$, which is a contradiction. Hence A and B do not satisfy the property (E-A).

III. MAIN RESULTS

Let F be the set of all fuzzy set on $X^2\times(0, \infty)$ that is $F = \{f: X^2\times(0, \infty)\to[0,1]\}$.

Definition 1

Let f and $g\in F$. The algebraic sum $f\oplus g$ of f and g is defined by
$f(x, y, t) \oplus g(x',y',t) = \sup_{t_1+t_2=t} \min\{f(x, y, t_1), g(x, y', t_2)\}$

Remarks 2

For every $x, y\in X$ and every $t>0$, we have
$f(x, y, 2t) \oplus f(x, y, 2t) \geq \min\{f(x, y, t), f(x, y, t)\} = f(x, y, t)$
ii) $f(x, y, t) \oplus 1 \geq \min\{f(x, y, t-\epsilon), f(x, x,\epsilon)\} = f(x,y,t-\epsilon)$
Letting $\epsilon\to 0$, we get $f(x,y,t)\oplus 1 \geq f(x,y,t)$.

Throughout this section Φ denotes a family of mappings such that for each $\varphi\in \Phi$, $\varphi:[0,1]^2\to[0,1]$ is continuous and increasing in each co-ordinate variable. Also $\gamma(t) = \varphi(t, t) \geq t$ for every $t\in[0,1]$.

Example 3

Let $\varphi: [0,1]^2\to[0,1]$ be defined by $\varphi(x, y) = \min\{x, y\}$.
Now, we prove our main result:

Theorem 4

Let A, B, S and T be mappings from a fuzzy metric space $(X, \mathcal{M}, *)$ into itself satisfying the following conditions:
(3.1) $A(X)\subseteq T(X)$, $B(X) \subseteq S(X)$

(3.2)

$$\mathcal{M}(Ax, By, t) \geq \varphi\left\{\begin{matrix}\mathcal{M}\left(Sx, Ty, \frac{2t}{k}\right), \mathcal{M}\left(Ax, Sx, \frac{2t}{k}\right) \oplus \mathcal{M}\left(By, Ty, \frac{2t}{k}\right), \\ \mathcal{M}\left(Ax, Ty, \frac{4t}{k}\right) \oplus \mathcal{M}\left(Sx, By, \frac{4t}{k}\right)\end{matrix}\right\}$$

for all x, y∈X, t>0, φ∈Φ, and 0 ≤ k< 2. Suppose that one of the pairs (A, S) and(B, T) satisfies the property (E, A). (A, S) and (B,T) are weakly compatible and one of A(X), B(X), S(X) and T(X) is a complete subspace of X. Then A, B, S and T have a unique common fixed point in X.

Proof: Suppose that the pair (B, T) satisfies the property (E-A). Then, There exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty}\mathcal{M}(Bx_n, z, t) = \lim_{n\to\infty}\mathcal{M}(Tx_n, z, t) = 1$ for some z∈X and all t>0. Therefore $\lim_{n\to\infty}\mathcal{M}(Bx_n, Tx_n, t) = 1$since $B(X) \subseteq S(X)$.

There exists a sequence $\{y_n\}$ in X such that $Bx_n = Sy_n$, hence $\lim_{n\to\infty}\mathcal{M}(Sy_n, z, t) = 1$.

We prove that $\lim_{n\to\infty}\mathcal{M}(Ay_n, z, t) = 1$. Using (3.2) we have

$$\mathcal{M}(Ay_n, Bx_n, t) \geq \varphi\left\{\begin{matrix}\mathcal{M}\left(Sy_n, Tx_n, \frac{2t}{k}\right), \mathcal{M}\left(Ay_n, Sy_n, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right), \\ \mathcal{M}\left(Ay_n, Tx_n, \frac{4t}{k}\right) \oplus \mathcal{M}\left(Sy_n, Bx_n, \frac{4t}{k}\right)\}\end{matrix}\right\}$$

$$= \varphi\begin{pmatrix}\mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right), \mathcal{M}\left(Ay_n, Bx_n, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right), \\ \mathcal{M}\left(Ay_n, Tx_n, \frac{4t}{k}\right) \oplus 1\end{pmatrix}$$

(3.3)

Since, $\liminf_{n\to\infty}\mathcal{M}\left(Ay_n, Bx_n, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right)$

$$\geq \liminf_{n\to\infty}\min\left\{\mathcal{M}\left(Ay_n, Bx_n, \frac{2t}{k} - \epsilon\right), \mathcal{M}(Bx_n, Tx_n, \epsilon)\right\}$$

$$= \liminf_{n\to\infty}\mathcal{M}\left\{Ay_n, Bx_n, \frac{2t}{k} - \epsilon\right\}$$

letting∈→0, in the above inequality, we get

$\liminf_{n\to\infty}\mathcal{M}\left\{Ay_n, Bx_n, \frac{2t}{k}\right\} \oplus \mathcal{M}\left\{Bx_n, Tx_n, \frac{2t}{k}\right\} \geq \liminf_{n\to\infty}\mathcal{M}\left(Ay_n, Bx_n, \frac{2t}{k}\right)$.

Also, by Remark 3..2,

$\liminf_{n\to\infty}\mathcal{M}\left(Ay_n, Tx_n, \frac{4t}{k}\right) \oplus \mathcal{M}\left(Sy_n, Bx_n, \frac{4t}{k}\right) = \liminf_{n\to\infty}\mathcal{M}\left(Ay_n, Tx_n, \frac{4t}{k}\right) \oplus 1 \geq \liminf_{n\to\infty}\mathcal{M}\left(Ay_n, Tx_n, \frac{2t}{k}\right)$,

hence letting n→∞, in inequality (3.3), we get,

$$\liminf_{n\to\infty}\mathcal{M}(Ay_n, z, t) = \mathcal{M}\left(\liminf_{n\to\infty}(Ay_n, z, t)\right) = \liminf_{n\to\infty}\mathcal{M}(Ay_n, Bx_n, t)$$

$$\geq \varphi \liminf_{n\to\infty}\mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right), \liminf_{n\to\infty}\left\{\mathcal{M}\left(Ay_n, Bx_n, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right)\right\}$$

$$\liminf_{n\to\infty}\left\{\mathcal{M}\left(Ay_n, Tx_n, \frac{4t}{k}\right) \oplus 1\right\} \geq$$

$$\varphi\left(1, \liminf_{n\to\infty}\mathcal{M}\left(Ay_n, Bx_n, \frac{2t}{k}\right), \liminf_{n\to\infty}\mathcal{M}\left(Ay_n, Tx_n, \frac{2t}{k}\right)\right)$$

$$\geq \varphi\left(\liminf_{n\to\infty}\mathcal{M}\left(Ay_n, Bx_n, \frac{2t}{k}\right), \liminf_{n\to\infty}\mathcal{M}\left(Ay_n, Bx_n, \frac{2t}{k}\right), \liminf_{n\to\infty}\mathcal{M}\left(Ay_n, Tx_n, \frac{2t}{k}\right)\right)$$

$$= \varphi\left\{\begin{matrix}\mathcal{M}\left(\liminf_{n\to\infty}Ay_n, z, \frac{2t}{k}\right), \\ \mathcal{M}\left(\liminf_{n\to\infty}Ay_n, z, \frac{2t}{k}\right), \mathcal{M}\left(\liminf_{n\to\infty}Ay_n, z, \frac{2t}{k}\right)\end{matrix}\right\}$$

$$\geq \mathcal{M}\left(\liminf_{n\to\infty}Ay_n, z, \frac{2t}{k}\right)$$

⋮

$\geq \mathcal{M}\left(\liminf_{n\to\infty}Ay_n, z, \left(\frac{2}{k}\right)^n t\right) \to 1$.

Similarly,

$\limsup_{n\to\infty}\mathcal{M}(Ay_n, z, t) = \mathcal{M}\left(\limsup_{n\to\infty}Ay_n, z, t\right) = 1$, hence, $\lim_{n\to\infty}\mathcal{M}(Ay_n, z, t) = 1$.

Assume that S(X) is a closed subset of X. Then, There exists u∈X such that Su=z using (3.2), we get

$$\mathcal{M}(Au, Bx_n, t) \geq \varphi(\mathcal{M}\left(Su, Tx_n, \frac{2t}{k}\right), \mathcal{M}\left(Au, Su, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right),$$

$$\mathcal{M}\left(Au, Tx_n, \frac{4t}{k}\right) \oplus \mathcal{M}\left(Su, Bx_n, \frac{4t}{k}\right))$$

$$= \varphi\begin{pmatrix}\mathcal{M}\left(z, Tx_n, \frac{2t}{k}\right), \mathcal{M}\left(Au, z, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right), \\ \mathcal{M}\left(Au, Tx_n, \frac{4t}{k}\right) \oplus \mathcal{M}\left(z, Bx_n, \frac{4t}{k}\right)\end{pmatrix} \quad (3.4)$$

In addition, it is easy to verify that

$\liminf_{n\to\infty}\mathcal{M}\left(Au, Su, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right) \geq \mathcal{M}\left(Au, Su, \frac{2t}{k}\right)$→(2.5).

In fact, for all $\epsilon \in \left(0, \frac{2t}{k}\right)$, we have

$$\mathcal{M}\left(Au, Su, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right) \geq \min\left\{\mathcal{M}\left(Au, Su, \frac{2t}{k} - \epsilon\right), \mathcal{M}(Bx_n, Tx_n, \epsilon)\right\}$$

Since $\lim_{n\to\infty}Bx_n = \lim_{n\to\infty}Tx_n = Su$, the above inequality implies that,

$$\liminf_{n\to\infty}\left(\mathcal{M}\left(Au, Su, \frac{2t}{k}\right) \oplus \mathcal{M}\left(Bx_n, Tx_n, \frac{2t}{k}\right) \geq \mathcal{M}\left(Au, Su, \frac{2t}{k} - \epsilon\right)\right)$$

Letting ∈→0, in the above inequality, we get (3.5). Also by Remark (3.2), we get

$$\liminf_{n\to\infty}\left(\mathcal{M}\left(Au, Tx_n, \frac{4t}{k}\right) \oplus \mathcal{M}\left(Su, Bx_n, \frac{4t}{k}\right)\right) = \liminf_{n\to\infty}\mathcal{M}\left(Au, Tx_n, \frac{4t}{k}\right) \oplus 1$$

$$\geq \liminf_{n\to\infty}\mathcal{M}\left(Au, Tx_n, \frac{2t}{k}\right)$$

$$= \mathcal{M}\left(\mathrm{Au}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right)$$

So, letting n→∞, in inequality (3.4), we get

$$\mathcal{M}(\mathrm{Au}, \mathrm{z}, \mathrm{t}) \geq \varphi\left(1, \mathcal{M}\left(\mathrm{Au}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right), \mathcal{M}\left(\mathrm{Au}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right)\right)$$

$$\geq \varphi\left(\mathcal{M}\left(\mathrm{Au}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right), \mathcal{M}\left(\mathrm{Au}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right), \mathcal{M}\left(\mathrm{Au}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right)\right)$$

$$\geq \mathcal{M}\left(\mathrm{Au}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right)$$

$$\vdots$$

$$\geq \mathcal{M}\left(\mathrm{Au}, \mathrm{z}, \left(\frac{2}{\mathrm{k}}\right)^{\mathrm{n}} \mathrm{t}\right) \to 1.$$

Hence, $\mathcal{M}(\mathrm{Au}, \mathrm{z}, \mathrm{t}) = 1$.

That is, Au= Su= z. Since, A(x)⊆T(x), There exists v∈X s.t z=Tv. Using (3.2), and Remark (3.2),

we have $\mathcal{M}(\mathrm{z}, \mathrm{Bv}, \mathrm{t}) = \mathcal{M}(\mathrm{Au}, \mathrm{Bv}, \mathrm{t})$

$$\geq \varphi\left(\mathcal{M}\left(\mathrm{Su}, \mathrm{Tv}, \frac{2\mathrm{t}}{\mathrm{k}}\right)\right),$$

$$\mathcal{M}\left(\mathrm{Au}, \mathrm{Su}, \frac{2\mathrm{t}}{\mathrm{k}}\right) \oplus \mathcal{M}\left(\mathrm{Bv}, \mathrm{Tv}, \frac{2}{\mathrm{k}}\mathrm{t}\right),$$

$$\mathcal{M}\left(\mathrm{Au}, \mathrm{Tv}, \frac{4\mathrm{t}}{\mathrm{k}}\right) \oplus \mathcal{M}\left(\mathrm{Su}, \mathrm{Bv}, \frac{4\mathrm{t}}{\mathrm{k}}\right)\Big)$$

$$= \varphi\left(1, 1 \oplus \mathcal{M}\left(\mathrm{Bv}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right), 1 \oplus \mathcal{M}\left(\mathrm{z}, \mathrm{Bv}, \frac{4\mathrm{t}}{\mathrm{k}}\right)\right)$$

$$\geq \varphi\left(\mathcal{M}\left(\mathrm{Bv}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right), \mathcal{M}\left(\mathrm{Bv}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right), \mathcal{M}\left(\mathrm{z}, \mathrm{Bv}, \frac{2\mathrm{t}}{\mathrm{k}}\right)\right)$$

$$\geq \mathcal{M}\left(\mathrm{Bv}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right)$$

$$\vdots$$

$$\geq \mathcal{M}\left(\mathrm{Bv}, \mathrm{z}, \left(\frac{2}{\mathrm{k}}\right)^{\mathrm{n}} \mathrm{t}\right) \to 1.$$

Hence *z= Bv= Tv*.

Since the pairs (*A*, *S*) and (*B*, *T*) are weakly compatible, we obtain *Az = Sz* and *Bz =Tz* using the inequality (3.2), we have

$$\mathcal{M}(\mathrm{Az}, \mathrm{z}, \mathrm{t}) = \mathcal{M}(\mathrm{Az}, \mathrm{Bv}, \mathrm{t})$$

$$\geq \varphi\begin{pmatrix} \mathcal{M}\left(\mathrm{Sz}, \mathrm{Tv}, \frac{2\mathrm{t}}{\mathrm{k}}\right), \mathcal{M}\left(\mathrm{Az}, \mathrm{Sz}, \frac{2\mathrm{t}}{\mathrm{k}}\right) \oplus \mathcal{M}\left(\mathrm{Bv}, \mathrm{Tv}, \frac{2\mathrm{t}}{\mathrm{k}}\right), \\ \mathcal{M}\left(\mathrm{Az}, \mathrm{Tv}, \frac{4\mathrm{t}}{\mathrm{k}}\right) \oplus \mathcal{M}\left(\mathrm{Sz}, \mathrm{Bv}, \frac{4\mathrm{t}}{\mathrm{k}}\right) \end{pmatrix}$$

$$= \varphi\left(\mathcal{M}\left(\mathrm{Az}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right), 1 \oplus 1, \mathcal{M}\left(\mathrm{Az}, \mathrm{z}, \frac{4\mathrm{t}}{\mathrm{k}}\right) \oplus \mathcal{M}\left(\mathrm{Az}, \mathrm{z}, \frac{4\mathrm{t}}{\mathrm{k}}\right)\right)$$

$$\geq \varphi\left(\mathcal{M}\left(\mathrm{Az}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right), \mathcal{M}\left(\mathrm{Az}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right), \mathcal{M}\left(\mathrm{Az}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right)\right)$$

$$\geq \mathcal{M}\left(\mathrm{Az}, \mathrm{z}, \frac{2\mathrm{t}}{\mathrm{k}}\right)$$

$$\vdots$$

$$\geq \mathcal{M}\left(\mathrm{Az}, \mathrm{z}, \left(\frac{2}{\mathrm{k}}\right)^{\mathrm{n}} \mathrm{t}\right) \to 1.$$

Then Az = Sz = z.

Similarly, we can prove that z = Bz =Tz. Therefore z is a common fixed point of A, B, S and T.

Uniqueness: Uniqueness easily follows from (3.2).

Corollary 5

Let *A* and *S* be mappings from a fuzzy metric space (X,M, *) into itself satisfying the following condition

1. A(X) ⊆ S(X)
2. $$M(\mathrm{Ax}, \mathrm{Ay}, \mathrm{t}) \geq \varphi\begin{pmatrix} M\left(\mathrm{Sx}, \mathrm{Sy}, \frac{2\mathrm{t}}{\mathrm{k}}\right), \mathrm{M}\left(\mathrm{AX}, \mathrm{Sx}, \frac{2\mathrm{t}}{\mathrm{k}}\right) \\ \oplus M\left(\mathrm{Ay}, \mathrm{Sy}, \frac{2\mathrm{t}}{\mathrm{k}}\right), \\ M\left(\mathrm{Ax}, \mathrm{Sy}, \frac{4\mathrm{t}}{\mathrm{k}}\right) \oplus M\left(\mathrm{Sx}, \mathrm{Ay}, \frac{4\mathrm{t}}{\mathrm{k}}\right) \end{pmatrix}$$

for all *x*, *y*∈*X*, and t>0, Where 0≤k< 2. Suppose that the pair (*A*,*S*) satisfies the property (E-A), (*A*,*S*) is weakly compatible and one of *A(X)* and *S(X)* is a complete subspace of *X*. Then A and S have a unique common fixed point in X.

IV. CONCLUSION

We prove common fixed point theorems for weakly compatible mappings satisfying strict contractive condition in fuzzy metric spaces by using property (E.A). Our results generalize many known fixed points results in various spaces.

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